The Peculiar Mathematical Properties of the Force Free Field Equation and the Solar X-Ray Corona:

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The force free field equation $\nabla \times \boldsymbol{B} = \alpha \boldsymbol{B}$ *has two families of complex characteristics and one family of real characteristics, namely the field lines, as may be seen from the divergence of the field equation,* $\boldsymbol{B} \cdot \nabla \alpha = 0$. *Consider a magnetic field extending generally in the z-direction through an infinitely conducting fluid from a rigid endplate at z = 0 to another rigid endplate at z = L. The magnetic flux that makes up the magnetic field is interlaced in some arbitrary manner as it extends from z = 0 to z = L. The question is the mathematical form of the equilibrium interlaced field. Note, then, that the field equation can be reduced to the mathematical form of the 2D vorticity equation. The problem is that only an interlacing topological set of measure zero is accommodated by the vorticity equation. But with all field lines connecting between fixed footpoints at z = 0 and z = L, it is obvious that all interlacing topologies have an equilibrium, regardless of whether the topology conforms to the vorticity equation. This brings us to the so called weak solutions, contain in surfaces of tangential discontinuity (TD's) lying along the real characteristics.They accommodate the remaining interlacing topologies, i.e. almost all interlacing topologies. That is to say, the equilibria of almost all interlacing topologies are interspersed with TD's. The field in the writhing columns between TD's satisfy the vorticity equation. It is suggested that this tendency to form TD's dissipates the free magnetic energy of the coronal X-ray filaments to provide the heat source for the X-ray corona. It is pointed out that energy requirements rule out wave heating of the X-ray corona, with only the quasi-static interlacing of the field lines by the solar granules providing enough magnetic free energy and the means to convert that free energy into heat.*